

An Investigation of the Positive Column of a Hollow Cathode Arc in a Magnetic Field. Part II

P. A. E. M. Janssen and F. Boeschoten *

Department of Electrical Engineering, Eindhoven University of Technology

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It is shown that the behaviour of the highly ionized and magnetized plasma in the positive column of the hollow cathode arc (described in Part I of this investigation) may be well explained with the macroscopic equations on basis of the MHD ordering ($eE_r \gg \Delta p/n_e$). In particular the rotation of the plasma column around its axis is properly described by an equation for the electrical potential.

Viscous effects (due to ion-ion collisions) play an important role in the transport of particles, momentum and energy. The strong shear in the plasma rotation goes along with an inwardly directed ion flux, oppositely to the "classical" outward ion flux (due to ion-electron collisions), thus leading to an improved confinement of the ions. As the electron viscosity is negligible, the electrons move outward with the "classical" velocity. The resulting non-ambipolar plasma transport leads to an inwardly directed current and the corresponding $\mathbf{j} \times \mathbf{B}$ force drives the plasma rotation against the friction of the ions (ion-ion and ion-neutral collisions).

The stationary power balance is well fulfilled by the ions, and within a factor two also by the electrons. Most of the power is handled by the electrons.

1. Introduction

There is still a lack of highly ionized and magnetized laboratory plasma which are known and understood in detail [1]. Due to its relative simplicity the positive column of a hollow cathode arc seems to offer a possibility to improve somewhat on this situation. In Part I of this investigation the main characteristics of a large sized ($L \gg \lambda$) hollow cathode arc were reported [2]. The plasma parameters n_e , T_i , T_e , \mathbf{E} and \mathbf{v} were measured as function of radius r and axial distance z . They were varied by changing the gas discharge parameters — arc current I , gas feed Q , magnetic field strength B , arc length L and core diameter d .

The plasma density $n_e \leq 2 \times 10^{14}$ part./cm³; the ion temperature T_i ranges from 1 eV to 30 eV. The plasma is strongly ionized (up to 98%) and a large part of the column is strongly magnetized ($\omega_{ci} \tau_i > 1$). The electrical conductivity is „normal“ and the ratio of induced to imposed magnetic field $\ll 1$.

In this Part (II) the two fluid equations of a plasma are used to explain the experimental findings. As it was found that the perpendicular

electrical force is much larger (about 5 times) than the pressure force, a MHD ordering is applied to the equations of motion. In this way the observed plasma motion \mathbf{v} may be properly expressed in terms of the other plasma parameters.

A striking property of the plasma column under investigation is the (strongly sheared) *rotation* around its axis. The cause of this rotation lies in a radial current and a corresponding $\mathbf{j}_r \times \mathbf{B}$ force which acts against the viscous friction forces. In contrast to what was assumed in earlier calculations of Klüber [3] the radial component of the ion-velocity is generally non-vanishing. Klüber's model is valid whenever the dimensionless number $R = (\omega_{ce} \tau_e)^2 / H_a^2 \ll 1$, where H_a^2 is the Hartmann number $L_\perp^2 \sigma_\parallel B^2 / \eta$. For the conditions prevailing in this study $R > 1$. In deriving the potential equation from the equations of motion, the MHD ordering is utilized consistently and it is shown that the rotation can be derived from the potential to the required order. For a simplified model an analytic solution of the potential may be given.

The radial transport of the ions is strongly influenced by the ion *viscosity*, which in the presence of the sheared rotation leads to an inward ion flux. This flux is oppositely directed to the outward transport due to friction between ions and electrons („classical diffusion“). As the viscosity of the electron fluid is negligible such a compensation does not occur in the radial electron flux. The total effect is an inwardly directed current which makes the plasma rotation possible (see above). Apparently

* Institut für Plasmaphysik der Kernforschungsanlage Jülich GmbH, D-5170 Jülich.

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Reprint requests to Dr. F. Boeschoten, Institut für Plasmaphysik der Kernforschungsanlage Jülich GmbH, 5170 Jülich.

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the perpendicular plasma transport is non-ambipolar.

With the plasma motion being known, the particle balance equation may be used to find an expression for the radial density distribution (which according to the measurements is approximately Gaussian). An exact treatment is impeded by a lack of precise knowledge of ionization- and recombination rates in the arc. A satisfactory solution may be found, however, under the assumption that they are negligible small [4].

The power balance is made up for the ions and for the electrons. It is well fulfilled for the ions. For the electrons the energy transport equations cannot be evaluated with the wanted precision, due to the errors in the measurements of the electron temperature and to the fact that some of the terms depend strongly on the temperature (parallel conduction $\sim T_e^{7/2}$!). For the middle section of the arc the stationary power balance of the electrons is fulfilled within a factor two. Taking the inaccuracies into consideration it cannot be excluded, however, that part of the energy is lost in perpendicular direction by the strong l.f. oscillations [2]. Due to the radial phase shift of these oscillations, they may cause a radial heat transport by the electrons in the presence of a gradient in the electron temperature [5].

2. Plasma Motion

2.1. Equations of Motion

The equations of motion for ions and electrons read (see e.g. Braginskii [6]):

$$m_s n_s \frac{d\mathbf{u}_s}{dt} = -\nabla p_s - \nabla \cdot \Pi_s + e_s n_s (\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) \pm \mathbf{R} - m_s \mathbf{u}_s Q_s. \quad (1)$$

Here s refers to electrons (e) or ions (i) and $d^{(s)}/dt = \partial/\partial t + \mathbf{u}_s \cdot \nabla$. Furthermore, n_s is the density, \mathbf{u}_s the macroscopic velocity, p_s the pressure, Π_s the stress tensor, \mathbf{R} is the friction force between electrons and ions, Q_s is the source term (recombination and ionization) in the continuity equation.

It follows from the measurements that in the plasma of the hollow cathode arc (see I, Fig. 9) the perpendicular electric force enE_\perp is much larger than the pressure gradient. The electrical drift velocity \mathbf{v}_E is of the order of the ion thermal

velocity $v_{th,i}$, so that a MHD ordering has to be used [7]:

$$enE_\perp = 0 \left(\frac{1}{\varepsilon} \nabla p \right). \quad (2)$$

Here $\varepsilon = r_{ci}/L_\perp$, where r_{ci} is the ion cyclotron radius and L_\perp is a typical gradient length in perpendicular direction. In the experiment discussed in Part I ε is of the order 1/5, depending on the experimental conditions.

Furthermore, the axial gradient length L_\parallel is evidently much larger than L_\perp , viz.

$$L_\perp/L_\parallel \ll \varepsilon, \quad (3)$$

and according to $\nabla \times \mathbf{E} = 0$, the order of magnitude of the parallel component of the electric field is

$$E_\parallel = 0 \left(\frac{L_\perp}{L_\parallel} E_\perp \right). \quad (4)$$

Finally, the inequality: $\omega_{ci}\tau_{i,i} \ll \omega_{ce}\tau_{e,i}$ holds.

Charge conservation requires for the current $\mathbf{j} = e n(\mathbf{u}_i - \mathbf{u}_e)$:

$$\nabla \cdot \mathbf{j} = \nabla_\perp \cdot \mathbf{j}_\perp + \nabla_\parallel \cdot \mathbf{j}_\parallel = 0. \quad (5)$$

To determine \mathbf{j}_\perp the perpendicular components of the momentum equations for ions and electrons are used. According to Eq. (2) the following ordering of the terms in the ion momentum equation applies: both components of the Lorentz force $-en(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ are of order 1 (times $en v_{th} B$), the inertia term and pressure gradient are of order ε , the viscosity is of order ε^2 and ion-electron friction is assumed to be order ε^2 ($\omega_{ce}\tau_{e,i} \gg 1$). Estimating Q_s from the equation of continuity it is found that the term $m_s \mathbf{u}_s Q_s$ is order ε^3 for the ions. Expanding the ion velocity in powers of the small parameter ε the ion momentum equation can be solved order by order and to second order:

$$\begin{aligned} \mathbf{u}_i = \mathbf{v}_E - \frac{\nabla p_i \times \mathbf{B}}{enB^2} - \frac{m_i}{e} \frac{D}{Dt} \frac{\mathbf{v}_E \times \mathbf{B}}{B^2} \\ - \frac{(\nabla \cdot \Pi_{i0} - \mathbf{R}_{i,e}) \times \mathbf{B}}{enB^2} - \frac{m_i}{eB^2} \\ \cdot \left. \frac{d\mathbf{u}}{dt} \right|_1 \times \mathbf{B}, \end{aligned} \quad (6)$$

where $\mathbf{v}_E = \mathbf{E} \times \mathbf{B}/B^2$, $D/Dt = \partial/\partial t + \mathbf{v}_E \cdot \nabla$ and $d\mathbf{u}/dt|_1$ is the rate of change of \mathbf{u}_i in first order. Moreover in $\nabla \cdot \Pi$ and $\mathbf{R}_{i,e}$ we have substituted the $\mathbf{E} \times \mathbf{B}$ drift \mathbf{v}_E because these terms are second order in ε .

Similarly we find for the electrons:

$$\mathbf{u}_e = \mathbf{v}_E + \frac{\nabla p_e \times \mathbf{B}}{enB^2} - \frac{\mathbf{R}_{e,i} \times \mathbf{B}}{enB^2}. \quad (7)$$

Since $m_e/m_i \ll 1$, electron inertia and viscosity can be neglected.

From (6) and (7) we obtain for the perpendicular component of the current

$$\mathbf{j}_\perp = \frac{\nabla(p_e + p_i) \times \mathbf{B}}{B^2} - \frac{\nabla \cdot \Pi_{i0} \times \mathbf{B}}{B^2} - m_i n \frac{D}{Dt} \frac{\mathbf{v}_E \times \mathbf{B}}{B^2} - \frac{m_i n}{B^2} \frac{d\mathbf{u}_i}{dt} \Big|_1 \times \mathbf{B}, \quad (8)$$

where the conservation of momentum in electron-ion collisions has been used: $R_{i,e} = -R_{e,i}$.

The parallel current j_\parallel is derived from the parallel component of the electron momentum equation:

$$0 = -\frac{\partial p_e}{\partial z} - enE_\parallel + R_\parallel, \quad (9)$$

where $R_\parallel = en\sigma_\parallel^{-1}j_\parallel - 0.71n\nabla kT_e$.

Using Eq. (3) and Eq. (5) the various terms in Eq. (9) are found to be: $0.71n\nabla kT_e = 0$ ($\varepsilon enE_\parallel$), $\partial p_e/\partial z = 0$ ($\varepsilon enE_\parallel$).

Thus:

$$j_\parallel = \sigma_\parallel E_\parallel + O(\varepsilon). \quad (10)$$

The accuracy of the approximation will be discussed later.

2.2. Potential Equation

For an axisymmetrical equilibrium in the stationary state the radial current is found to be:

$$j_r = -(\nabla \cdot \Pi_{i0})_\varphi / B. \quad (11)$$

Substitution of Eq. (10) and (11) in Eq. (5) yields

$$-\nabla_\perp \cdot \{(\nabla \cdot \Pi_{i0})_\varphi / B\} + \nabla_\parallel \cdot \sigma_\parallel E_\parallel = 0. \quad (12)$$

Obviously, in the stationary state the perpendicular component of the current is due to ion-ion collisions whereas the parallel component of the current is determined by electron-ion collisions. In the plasma under investigation the contributions to $(\nabla \cdot \Pi_{i0})_\varphi$ are mainly due to the sheared electrical drift velocity $v_E(r)$; in our approximation the other terms in the ion viscosity are of $O(\varepsilon^3)$ and are neglected. Writing the expression for the stress tensor given by Braginskii [6] in cylindrical coordinates we obtain with $\mathbf{E} = -\nabla\varphi$, the potential

equation:

$$\frac{1}{r} \frac{\partial}{\partial r} \left[\frac{1}{r} \frac{\partial}{\partial r} \left\{ r^3 \eta_1 \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \varphi}{\partial r} \right) \right\} \right] - \frac{\partial}{\partial z} \sigma_\parallel B^2 \frac{\partial \varphi}{\partial z} = 0, \quad (13)$$

where η_1 is the ion-viscosity coefficient (cf. [6] Eq. (2.23)).

Since j_\perp is calculated to second order and j_\parallel to zeroth order in ε the consistency of this derivation has to be considered. Requiring the first and second term in Eq. (13) of the same order, we obtain

$$\frac{L_\perp^4}{L_\parallel^2} \frac{\sigma_\parallel B^2}{\eta_1} = O(1).$$

For the experiment considered here this condition is obeyed ($L_\perp \cong 10^{-2}$ m, $L_\parallel \cong 1$ m, $\sigma_\parallel \cong 2 \cdot 10^4$ mho m⁻¹, $\eta_1 = 10^{-5}$ Nsm⁻², $B = 1/3$ T).

2.3. Solution for a Uniform Plasma

For a uniform plasma (n , T_i and T_e constant)* the potential equation may be written in a concise form:

$$\Delta_\perp^2 \varphi - \frac{1}{l^2} \frac{\partial^2}{\partial z^2} \varphi = 0, \quad \Delta_\perp = \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} \quad (14)$$

where $l^2 \equiv \eta_1/\sigma_\parallel B^2$.

In order to solve Eq. (14) the boundary conditions have to be specified.

Consider the simplified case of an uniform plasma in a configuration given in Figure 1. In this figure the lines are supposed to be non-conducting walls

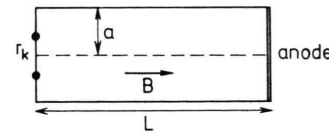


Fig. 1. Model of the plasma boundaries; $r_k = d/2$ is the radius of the cathode tube ($\neq r_k$ of Part I).

and the dots represent the cathode, which is a ring of infinitesimal thickness. The endplate on the right is a conducting anode. The boundary conditions then read

$$\begin{aligned} 1) & E_r = 0, \quad j_r = 0 \quad \text{for } r = 0, \quad r = a, \\ 2) & r = 0, \quad j_z = (I_0/2\pi r_k) \delta(r - r_k), \end{aligned} \quad (15)$$

I_0 is the current,

$$z = L, \quad E_r = 0.$$

* In case $\omega_{ci}\tau_{i,i} < 1$ Eq. (13) is also valid for a non-uniform density since then $\eta_1 = nkT_i\tau_{i,i}$ is independent of n .

The boundary value problem (14), (15) can be solved with the method of separation of variables and in a good approximation we obtain (see Ref. [8]),

$$\varphi \cong \varphi_0 + \frac{I_0}{\pi a^2 \sigma_{\parallel}} \times \left[z - \frac{J_0(\nu_1 r) J_0(\nu_1 r_k) \sinh \alpha(L - z)}{\alpha J_0^2(\nu_1 a) \cosh \alpha L} \right], \quad (16)$$

a solution which is valid away from the cathode region. Here $\nu_1 = j_{1,1}/a$ ($j_{1,1}$ is the first zero of the Bessel function of the first kind and first order) and $\alpha = l \nu_1^2$.

In lowest order of ε the angular frequency Ω is given by the $\mathbf{E} \times \mathbf{B}$ -drift frequency, hence

$$\Omega = \frac{u_{\varphi}}{r} \cong -\frac{1}{r} \frac{E_r}{B} = \frac{1}{r B} \frac{\partial \varphi}{\partial r}, \quad (17)$$

and substitution of the approximate expression for φ (Eq. (16)) yields

$$\Omega \cong \frac{\nu_1 I_0}{\pi a^2 \sigma_{\parallel} \alpha} \times \frac{J_1(\nu_1 r)}{r B} \times \frac{J_0(\nu_1 r_k) \sinh \alpha(L - z)}{J_0^2(\nu_1 a) \cosh \alpha L}. \quad (18)$$

For „standard” discharge parameters ($L = 1$ m; $I = 100$ A; $r_k = 6.5$ mm; $B = 1/3$ T) and $a = 4 r_k$, the r - and z -dependence of the angular frequency Ω is shown in Fig. 2a and Fig. 2b ($\eta_i = 10^{-5}$ Ns m $^{-2}$; $j_{1,1} = 3.8$; $\sigma_{\parallel} = 2 \times 10^4$ mho m $^{-1}$ for $T_e(0,50)$).

Though a highly schematic plasma configuration is used, the agreement with experiment is reasonable. Note that this model only applies to the core region of the plasma column.

Substitution of Eq. (16) in Eq. (11) yields the radial component of the current,

$$j_r = -\nu_1^2 \frac{\eta_1}{B^2} \frac{\partial \varphi}{\partial r}. \quad (19)$$

If we write formally $j_r = “\sigma_{\perp}” E_r$ we find “ σ_{\perp} ” = $\nu_1^2 \eta_1 / B^2 \cong 2$ mho m $^{-1}$.

Thus it turns out that in a good approximation the current can be derived from the plasma potential for a uniform plasma. In the experiment of Part I, “ σ_{\perp} ”/ $\sigma_{\parallel} = 10^{-4}$.

For more details and for the case of a non-uniform plasma we refer to [8].

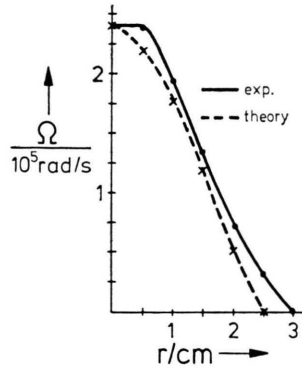


Fig. 2a. Comparison of experimental and theoretical radial dependence of Ω ($z = 50$ cm).

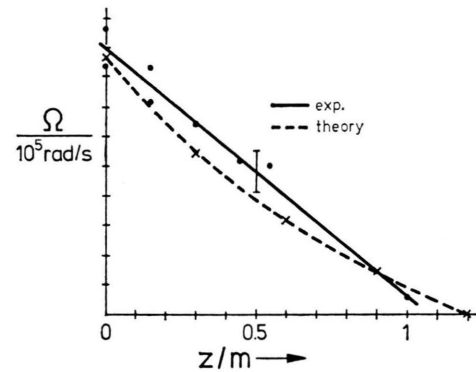


Fig. 2b. Comparison of experimental and theoretical axial dependence of Ω ($r = 0$ cm).

3. Particle Balance

As mentioned in [2] the plasma particles which are produced in the cathode region move with a velocity of about 6×10^2 m/s in the direction of the anode where the larger part recombines. Consequently they stay in the positive column for a time $\lesssim \tau_{\parallel} \simeq 2 \times 10^{-3}$ s. The question is whether the radial transport of the plasma particles may manifest itself during this time. A direct demonstration that this is the case is given by the plasma which is present around the core of the arc. However, the (visible) divergence of the core of the arc puts an upper limit to the radial velocity; next to the core $v_r \lesssim 1.2$ m/s.

A lower limit of the average confinement time $\bar{\tau}_{\text{conf}}$ of the particles may be estimated from the integral conservation law. Assuming that all the particles which enter the cathode region are ionized:

$$N_{\text{tot}} = \int_{V_{\text{tot}}} n_e dV \cong Q \bar{\tau}_{\text{conf}}.$$

The volume integral is taken over the plasma of the positive column. Ionization and recombination in this region are neglected. Under standard conditions ($B = 0.34 T$; $Q = 4.5$ cc NTP/s, $q^2 = 6.4 \times 10^{-4} \text{ m}^2$, $n_e(0) = 1.8 \times 10^{20} \text{ m}^{-3}$, $N_{\text{tot}} = \pi q^2 \times L n_e(0) \cong 3.6 \times 10^{15}$ particles, $Q \cong 1.2 \times 10^{20}$ part/s and $\bar{\tau}_{\text{conf}} \cong 2 \times 10^{-3}$ s. This is about the same value as found for $\bar{\tau}_{\parallel}$. As $1/\bar{\tau}_{\text{conf}} = 1/\bar{\tau}_{\parallel} + 1/\bar{\tau}_{\perp}$ it may be concluded within the accuracy of this estimate that $\bar{\tau}_{\perp} \gtrsim 2 \times 10^{-3}$ s.

The so-called ‘‘classical perpendicular diffusion coefficient’’*:

$$D_{\perp}(\text{class}) = 3 \times 10^{-22}$$

$$\times \frac{n_e [\text{m}^{-3}] (T_e + T_i) [\text{eV}]}{B^2 [\text{T}] T_e^{3/2} [\text{eV}]} \text{ m}^2/\text{s}$$

varies somewhat along the axis; its average value is about $0.5 \text{ m}^2/\text{s}$. This leads to an average confinement time $\bar{\tau}_{\perp} \cong (q/2.4)^2 D_{\perp}$ of about 2×10^{-4} s which is one order of magnitude smaller (!) than the estimated lower limit. Likewise the radial velocity of the plasma, $v_r \lesssim 2 \text{ m/s}$, is found to be much smaller than calculated from $v_r \cong (2r/q^2) D_{\perp} \cong 20 \text{ m/s}$. Apparently the perpendicular transport of the ions occurs at a much lower speed than what is expected if only ‘‘classical diffusion’’ (which has its origin in collisions between electrons and ions) is operative.

The solution to this problem lies in the ion viscosity, which leads in the presence of the strongly sheared mass rotation to an inwardly directed ion flux which opposes the outwardly directed ion flux due to the radial pressure gradient. For the electron flux such a compensation does not occur, so that an inwardly directed current results which was discussed in the previous section.

A quantitative treatment must start from the particle conservation equation, which reads in the stationary state:

$$\nabla \cdot n_i \mathbf{u}_s = \beta_i n_i - \alpha n_i^2, \quad (20)$$

where α is the radiative recombination coefficient and $\beta_i n_i$ the production rate of charged particles by collisional ionization.

The right hand side of Eq. (20) is not well known. An approximate expression for β_i [9] yields in the middle of the core ($n_n \cong 10^{12} \text{ part/cm}^3$ and $T_e \cong 6 \text{ eV}$) $\beta_i \cong 5 \times 10^2 \text{ s}^{-1}$ and $\tau_{\text{ion}} \cong 2 \times 10^{-3}$ s. As this is comparable to τ_{conf} , τ_{\parallel} and τ , the con-

tribution of ionization may not be neglected. The radiative recombination coefficient, α , is even less well known; experiments with argon yield values of $\alpha \cong 10^{-11} \text{ cm}^3/\text{s}$. In the core region the ‘‘recombination time’’ for a particle $\tau_{\text{rec}} \cong \tau_{\text{ion}}$; collisional ionization and radiative recombination may balance each other approximately (Corona equilibrium). A treatment of this subject is given in von der Sijde et al. [10].

In order to evaluate the left hand side of Eq. (20) the particle fluxes have to be determined. From Eqs. (6) and (7) we obtain for the radial particle flux of ions and electrons:

$$n_e u_{ir} = \frac{1}{r^2} \frac{\partial}{\partial r} \frac{r^3 \eta_1}{e B} \frac{\partial}{\partial r} \Omega_E + n u_{\text{amb}}, \quad (24)$$

$$n_e u_{er} = n u_{\text{amb}},$$

where $n_e u_{\text{amb}} = -1/e B \omega_{ce} \tau_{e,i}$

$$\times \left[\frac{\partial}{\partial r} (p_e + p_i) - m_i n r \Omega_E^2 - \frac{3}{2} n \frac{\partial}{\partial r} k T_e \right].$$

The first term in the expression for the ion-particle flux is already discussed in the context of the potential equation. This flux contribution is connected with an inwardly directed current as can be seen from Equation (19). The ambipolar velocity u_{amb} is due to electron-ion collisions and is usually directed outwards. The first term on the right hand side of the expression for $n u_{\text{amb}}$ is due to the plasma pressure gradient, the second term is due to the centrifugal force, and the last term is due to the dependence of the electron-ion collision frequency on T_e which leads to an extra force in azimuthal direction (Nernst effect). The ratio of the viscous flux to the ambipolar flux is approximately:

$$0.2 (m_i/m_e)^{1/2} \times (\partial \Omega / \partial r) / (r \omega_{ci} / q^2).$$

The contribution of parallel transport in the ion-continuity equation is rather small because of $L_{\perp}/L_{\parallel} \ll 1$. For electrons the parallel transport is of importance since the parallel electron velocity is large. A solution of Eq. (20) with neglect of the right hand side is given in [4].

In concluding this section it may be noted that it is surprising that up to $r = 4 \text{ cm}$ the plasma loss is within a factor two of what the ‘‘classical’’ theory predicts, though the low frequency oscillations cause very large periodic displacements of the plasma: $\Delta r \cong 0.6 \text{ cm}$ (see section 8 of [1]). Apparently this is due to the circumstance that the oscilla-

* As rightly pointed out by Braginskii [6], the word ‘‘diffusion’’ should be replaced by ‘‘leak rate’’.

tions in the plasma potential, Φ , and in the plasma density n_e , are found to be in phase. The time average of the radial particle flux due to the alternating electric drift is expected to be:

$$\frac{n_e(r, z) \Phi(r, z)}{2rB} \cos\left(\varphi_n - \varphi_\Phi + \frac{\pi}{2}\right) = 0$$

for $\varphi_\Phi = \varphi_n$.

4. The Power Balance

The total power which is fed to the arc follows directly from the (I, V) characteristic (Part I, Fig. 7) and is for operation under standard conditions ($I = 100$ A; $V \cong 70$ V) about 7 kW. Practically all this power is finally removed from the system by the water cooling of the electrodes and the wall. The question where and how this power flows is solved differently in the three regions of the discharge — the cathode region ($\Delta V \cong 45$ V), the positive column ($\Delta V \cong 15$ V) and the anode region ($\Delta V \cong 10$ V).

In the *cathode region* electrons which emerge from the hot glowing cathode are accelerated in a voltage difference of about 45 V, which is an optimum value for ionization of neutral argon atoms. Depending on the acquired velocity and the place where they are created, the ions are either accelerated in the direction of the cathode (which is heated by them) or are dragged with the electrons in the direction of the anode with a velocity of about 6×10^2 m/s (Part I, Figure 9). With Stephan-Boltzmann's law and the measured temperature profile along the cathode tube it is found that the black body radiation of the cathode amounts to about 0.8 kW. The heat conduction along the cathode tube amounts to about one tenth of this amount, so that all together roughly 1 kW is lost at the cathode tube. For ionization ($\chi_i = 15.75$ eV) and heating ($T_i = 8$ eV; $T_e = 12 - 16$ eV) of a flow $Q = 4.5$ cm³ NTP/s argon gas 0.8 kW power is needed. Thus of the 4.5 kW power which is used in the cathode region, about 3.5 kW flows into the adjacent region of the positive column; 0.8 kW of this amount is invested in the plasma particles and the rest flows as heat into the positive column.

At the other side of the discharge (where $T_e \cong 2$ eV and $T_i \cong 1$ eV) electrons which are accelerated in the *anode region* deliver about 1 kW power at the anode ($\Delta V \cong 10$ V). The heat flow from the positive column to the anode amounts to about 0.5 kW.

Recombination of particles which flow toward the anode ($v_z \cong 6 \times 10^2$ m/s) contribute to another 0.5 kW. All together about 2 kW power has to be removed from the anode by watercooling. About 4 kW power reaches the wall via the plasma.

The decrease of ion temperature, T_i , and electron temperature, T_e , towards the anode, makes it useful to distinguish in the positive column in axial direction three regions of 0.4 m length for which in the core region the parameter $\omega_{ci} \tau_{i,i}$ is respectively larger, about equal and smaller than one. Here, only the middle region where $\omega_{ci} \tau_{i,i} \cong 1$ is considered. In this region the mean free path length of the ions is smaller than the density gradient length; thus the macroscopic energy equation may be applied, whereas in the region near the cathode where ($\omega_{ci} \tau_{i,i} > 1$) this is questionable since there the ratio of Larmor radius to density gradient is of order one.

4.1. Power Balance of the Plasma in the Positive Column

For each species of particles the energy transport equation for a plasma is (see [6]):

$$\frac{\partial}{\partial x_\beta} \left\{ \left(\frac{n m}{2} u^2 + \frac{5}{2} n k T \right) u_\beta + q_\beta \right. \\ \left. + (II_{\alpha\beta} \cdot u_\beta) \right\} - e n \mathbf{E} \cdot \mathbf{u} - \mathbf{R} \cdot \mathbf{u} - Q = 0. \quad (22)$$

The tensor in the brackets of Eq. (22) represents the sum of the total energy transport with velocity \mathbf{u} (convection), the heat flux \mathbf{q} (conduction) and the work done by the total pressure forces ($II_{\alpha\beta}$ is the stress tensor). The other terms represent respectively the Joule dissipation, the momentum transfer between ion and electrons and the heat Q , generated in a gas of particles of a given species. The values of the various terms of Eq. (22) integrated over the middle section of the positive column are shown in Table 1 for ions and electrons.

The numbers are only approximate. Neither the theory nor the experiments are more accurate than a factor two. It is evident that:

- the electrons handle a much larger part of the energy transport than the ions,
- for the ions the losses and the gains are in balance within the accuracy of the various contributions; the contribution of the perpendicular heat condition is small due to the constancy of T_i with radius,

Table 1. Various contributions to the global power balance of ions and electrons in Watt.

Integrated value	Name	Ions	Electrons
$\frac{5}{2} nkT u_z$	parallel convection	-50	-250
q_z	parallel conduction	-2	-600
$\frac{5}{2} nkT u_r$	perpendicular convection	65	325
q_r	perpendicular conduction	<30	10
$\eta_{\perp} r \frac{\partial \Omega}{\partial r} u_{\varphi}$	viscosity	120	0
$e n \mathbf{E} \cdot \mathbf{u}$	Joule heat	60	-400
$\mathbf{R} \cdot \mathbf{u}$	i, e friction	-10	200
Q	i, e heat transfer	-200	0
	charge exchange	15	0
Σ	Sum	$\cong 0$	-700

$$\begin{aligned}
 z &= 0.4-0.8 \text{ m}, & \Delta V &= 4 \text{ V}, \\
 T_i &= 4-2 \text{ eV}, & n &= 2 \times 10^{20} \text{ m}^{-3}, \\
 T_e &= 8.5-5 \text{ eV},
 \end{aligned}$$

— the losses for the electrons are mainly due to perpendicular convection; the gains exceed the losses by a factor two, which is also the accuracy of the quoted numbers.

4.2. The Power Balance of the Ions

The perpendicular heat coefficient of the ions is relatively large and it is easy to show that lest the term q_{β} becomes much larger than the other terms of Eq. (22) it is required that $(1/r) \partial/\partial r (r q_{ir}) \rightarrow 0$. As the heat sources for the ions are negligible $q_{ir} \rightarrow 0$ which leads directly to the observed constancy of T_i with radius (Part I, Eq. (1)). Viscosity, Joule heat and perpendicular convection are of equal importance for the energy loss of the

ions. Most of the energy gain of the ions results from collisions with electrons.

4.3. The Power Balance of the Electrons

As is already pointed out in Sect. 3 the electrons move in radial direction with the ambipolar speed whereas the ions move much slower. Estimating the electron velocity for the middle region ($\omega_{ci} \tau_{i,i} \cong 1$) we obtain $u_{er} \cong 30 \text{ m/s}$ for $n_e = 2 \times 10^{20} \text{ m}^{-3}$ (the same order of magnitude is also found from the continuity equation).

By concluding this section the effect of the electric field may be noted. Due to the presence of the electric field the ions move outwards with a speed slower than the ambipolar speed and hence the ions are rather well confined. The electrons are transported in radial direction with the much higher ambipolar speed and since they handle a large part of the energy transport, they are responsible for the energy losses at the wall. Taking the inaccuracies into consideration it cannot be excluded that part of the energy is lost in perpendicular direction by l.f. oscillations. Due to the radial phase shift of these oscillations, they may cause a radial heat transport by the electrons in the presence of a temperature gradient ∇T_e [5].

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